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Monitoring the change of spatial discrete control Radix based on multi-scale wavelet transform algorithm

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Abstract. In order to improve effectiveness of vibration damage monitoring result for buildings, a kind of vibration damage monitoring algorithm for buildings based on multi-scale wavelet transform of spine space for two-dimensional discrete convolution of cardinal number was proposed. Firstly, aiming at influence of vibration data trend on detection result, gradient vector of wavelet transform was established so as to realize multi-scale detection of signal with different frequencies; then, in combination with polynomial spline function, algorithm of binary wavelet transform for two-dimensional discrete was expressed in the form of discrete convolution so as to simplify algorithm expression for improvement of data analysis effect; finally, effectiveness of algorithm was verified through simulation experiment.

Key words. Two-dimensional discrete convolution Spline space Multi-scale Wavelet transform; Buildings Damage monitoring.

1. Introduction

Large-scale structure may be damaged and degraded due to various factors in the process of long-term use. If there are problems not being immediately found and necessary measures are not taken, they may lead to serious accidents, life loss, and property loss. Therefore, it is very important to conduct proper evaluation for health of structure. Damage identification is a critical technology in structure health detection. Changes of structural dynamic characteristics are used for identification in civil engineering, mechanical engineering, aeronautical engineering, and other fields, which has been a research direction with wide attention.

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In recent years, the damage detection method based on vibration test has been more and more widely used in the field of civil engineering. The method refers to using signal processing and damage diagnosis technique to determine size and location of potential overall performance degradation or local damage for structure through obtained dynamic signal of structure. Its core idea is that modal parameter (inherent frequency, modal shape, and damping) is function of structural physical parameter (mass, rigidity, and damping), which can be used to judge damage of structure through looking for changes of dynamic characteristic of certain structure before and after structural damage. It belongs to structural non-destructive testing technology with simple, fast, and non-destructive advantages, and significant economic benefit and social benefit.

Improvement of vibration damage monitoring effect for buildings on the basis of wavelet analysis algorithm is realized in the thesis. Meanwhile, in order to improve performance of wavelet analysis algorithm, gradient vector of wavelet transform is established. In combination with polynomial spline function, algorithm of binary wavelet transform for two-dimensional discrete was expressed in the form of discrete convolution so as to simplify algorithm expression for improvement of data analysis effect

2. Multi-scale wavelet transform of spine space for two-dimensional discrete convolution of cardinal number

2.1. Establishment of gradient vector

In general, amplitude variation of objects along the edge is slow; amplitude variation of objects vertical to the edge is large. In addition, as size of objects is different, their edges have different scales. Under the condition of two-dimension, edge detection algorithm is used to calculate gradient vector of vibration signal $f(x, y)$ for building:

$$\Delta f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) . \tag{1}$$

Local maximum value of model is used to look for spatial position of vibration edge for buildings. Direction of gradient vector indicates the fastest direction of changes for vibration grey value of buildings. In order to calculate two partial derivatives of vibration signal for buildings, two-dimensional wavelet with two directivities are required. They can be separately taken as partial derivative of smooth function $\theta(x, y)$

$$\psi^x(x, y) = -\frac{\partial \theta(x, y)}{\partial x} \quad \psi^y(x, y) = -\frac{\partial \theta(x, y)}{\partial y} . \tag{2}$$

Integral of $\theta(x, y)$ on the plane of $x - y$ is 1 and is rapidly converged to 0. We can command:

$$\psi_j^x(x, y) = 2^{-j} \psi^x(2^{-j}x, 2^{-j}y) \quad \psi_j^y(x, y) = 2^{-j} \psi^y(2^{-j}x, 2^{-j}y) . \tag{3}$$

And define two components of wavelet transform:

$$W^x f(2^j, x, y) = (f(u, v), \psi_j^x(u - x, v - y)) = f * \psi_j^x(-x, -y)\,.$$

$$W^y f(2^j, x, y) = (f(u, v), \psi_j^y(u - x, v - y)) = f * \psi_j^y(-x, -y)\,. \tag{4}$$

Define of binary wavelet transform for arbitrary $f \in L^2(R^2)$ is shown as the following function family:

$$Wf(2^j, x, y) = \left\{W^x f(2^j, x, y), W^y f(2^j, x, y)\right\}_{j \in Z}\,. \tag{5}$$

In order to ensure completeness and stability of wavelet transform, the following conditions has to be met: there are twp positive constant A and B for $\forall(w_x, w_y) \in R^2 - \{(0,0)\}$ which makes

$$A \leq \sum_{j=-\infty}^{\infty} \left|\widehat{\psi}^x(2^j w_x, 2^j w_y)\right|^2 + \left|\widehat{\psi}^y(2^j w_x, 2^j w_y)\right|^2 \leq B\,. \tag{6}$$

$\widehat{\psi}^x$ and $\widehat{\psi}^y$ in above equation separately indicates Fourier transform of $\psi^x$and ψ^y. $\{\psi^x, \psi^y\}$ which meets above equation is called as binary wavelet. As $\{\psi^x, \psi^y\}$ is first-order partial derivative of smooth function $\theta(x, y)$, thus two components of two-dimensional binary wavelet transformation is equal to two components of gradient vector for signal $f(x, y)$ after being subject to smoothness.

$$\begin{pmatrix} W^x f(2^j, x, y) \\ W^y f(2^j, x, y) \end{pmatrix} = 2^j \begin{pmatrix} \frac{\partial}{\partial x}(f * \overline{\theta_j})(x, y) \\ \frac{\partial}{\partial y}(f * \overline{\theta_j})(x, y) \end{pmatrix} = 2^j \nabla (f * \overline{\theta_j})(x, y)\,. \tag{7}$$

Scale $S = 2^j$ in above equation is called as binary scale. When dimension S is large, convolution between signal and wavelet function eliminate the smaller change in signal, thus only larger upheaval points can be detected by low-frequency signal in wavelet decomposition. Therefore, in terms of value S with different size, upheaval points under different dimensions can be obtained, which means that multi-scale edge detection is used to detect signal with different frequency after wavelet decomposition. It is known from the above that direct ratio of model for gradient vector $\nabla(f * \overline{\theta_j})(x, y)$:

$$Mf(2^j, x, y) = \sqrt{|W^x f(2^j, x, y)|^2 + |W^y f(2^j, x, y)\,.|^2}\,. \tag{8}$$

Binary wavelet transformation can be used to realize multi-scale edge detection so as to solve local maximum in Equation (8); Equation (9) indicates direction of edge.

2.2. Cardinal spine space

Cardinal spine space refers to polynomial spline function space with equidistant nodes. It definition is as follows: in terms of each integer m, there is m-order cardinal spine space S_m which meet aggregation of the following conditions:

(1) Function $f \in C^{m-2}(R)$

(2) Algebraic polynomial with no more than $m-1$ times in arbitrary interval $(k, k+1), k \in Z$

Now, definition and some properties of B-spine can be discussed. First-order cardinal B-spine $N_1(t)$ is characteristic function on unit interval $[0,1)$. At the time of $m \geq 2$, recursive definition of $N_m(t)$ with convolution is:

$$\begin{aligned} N_m(t) :&= (N_{m-1} * N_1)(t) = \int_{-\infty}^{\infty} N_{m-1}(t-u) N_1(u) du \\ &= \int_0^1 N_{m-1}(t-u) du \end{aligned} \tag{9}$$

It is known in Equation (11) that first-order B- spine $N_1(t)$ of is piecewise constant. It can be known from definition of cardinal B-spine that:

$$N_m(t) = \int_0^1 N_{m-1}(t-u) du = \int_{t-1}^{t} N_{m-1}(u) du\,. \tag{10}$$

Piecewise expression of $N_2(t)$, $N_3(t)$, and $N_4(t)$ are easy to be solved in the way. For example, expression of $N_2(t)$ can be solved as follows:

$$N_2(t) = \int_{t-1}^{t} N_1(u) du = \int_{t-1}^{t} \chi_{[0,1)}(u) du\,. \tag{11}$$

Next, two-scale relationship of cardinal B-spine will be discussed. Firstly, $N_1(t)$ shall be subject to Fourier transformation so as to obtain:

$$\begin{aligned} \hat{N}_1(w) &= \int_{-\infty}^{\infty} N_1(t) e^{-jwt} dt = \int_0^1 e^{-jwt} dt = \frac{1}{jw} - \frac{1}{jw} e^{-jw} \\ &= e^{-jw/2} \left(\sin \frac{w}{2} \Big/ \frac{w}{2} \right) . \end{aligned} \tag{12}$$

Then, it shall be subject to $N_m(t) = (\underbrace{N_1 * N_1 * N_1 .... * N_1}_{m?})(t)$ so as to obtain:

$$\hat{N}_m(w) = \left(\hat{N}_1(w) \right)^m = e^{-jmw/2} \left(\sin \frac{w}{2} \Big/ \frac{w}{2} \right)^m . \tag{13}$$

Finally, relationship between $\hat{N}_m(w)$ and $\hat{N}_m(w/2)$ under different scales can be obtained as follows:

$$\hat{N}_m(w) = P(z) \hat{N}_m(w/2)\,. \tag{14}$$

Where $P(z) = \left(\frac{1+z}{2}\right)^m = \frac{1}{2^m}\sum_{k=0}^{m}\begin{pmatrix} m \\ k \end{pmatrix} z^k$ and $z = e^{-jw/2}$.

2.3. Two-dimensional discrete convolution wavelet

There are many methods to establish wavelet $\psi(x)$ with smooth function $\vartheta(x)$. Spine function has great smooth and edge extraction capacities. Therefore, four-order B-spine is used in the thesis as smooth function $\vartheta(x)$. It can be known that wavelet $\psi(x)$ is three-order spline function, thus three-order B-spline $N_3(x)$ shall be taken as corresponding scaling function. It can be known from previous discussion that support interval $N_3(x)$ of is $[0, 3]$. Central point $N_3(x)$ in wavelet $\psi(x)$ can be taken as center, which is 1.5. In view of above-mentioned, derivative $\vartheta'(x)$ of smooth function shall be selected, which is $N_4'(2x-1)$ shall be considered as wavelet function. Definition of cardinal spline:

$$N_m(x) := (N_{m-1} * N_1)(x) = (\underbrace{N_1 * N_1 * N_1 .... * N_1}_{m})(x)\,. \tag{15}$$

Time-domain convolution will multiply in the frequency, thus $N_3(x) = (N_1 * N_1 * N_1)(x)$ shall be subject to Fourier transformation:

$$\hat{N}_3(w) = \left(\hat{N}_1\right)^3 = \left(\frac{1 - e^{-jw}}{jw}\right)^3 . \tag{16}$$

It is known from relationship of two scales in Equation (16) that:

$$\hat{N}_3(w) = P(e^{-jw/2})\hat{N}_3(\frac{w}{2})\,. \tag{17}$$

Thus it can solve:

$$P(e^{-jw/2}) = \left(\frac{1 - e^{-jw}}{jw}\right)^3 . \tag{18}$$

As $N_4'(x) = \frac{N_3(x) - N_3(x-1)}{x-(x-1)} = N_3(x) - N_3(x-1)$, wavelet $\psi(x) = N_4'(2x-1) = \frac{N_3(2x-1) - N_3(2x-2)}{(2x-1)-(2x-2)} = N_3(2x-1) - N_3(2x-2)$ shall conduct Fourier transformation for above-mentioned equation:

$$\begin{aligned}\hat{\psi}(w) &= \frac{1}{2}\hat{N}_3(\frac{w}{2})e^{-jw/2} - \frac{1}{2}\hat{N}_3(\frac{w}{2})e^{-jw} \\ &= \frac{1}{2}\hat{N}_3(\frac{w}{2})(e^{-jw/2} - e^{-jw})\,.\end{aligned} \tag{19}$$

In case $z = e^{-jw/2}$ is substituted into scaling transformation equation (21) so as to infer

$$H(z) = \frac{1}{8}(1+z)^3 = 0.125 + 0.375z + 0.375z^2 + 0.125z^3\,. \tag{20}$$

In case $L(e^{-jw/2}) = \frac{1}{2}e^{-jw/2}(e^{-jw/2} - e^{-jw})$, $z = e^{-jw/2}$ shall be substituted into

$$L(z) = 0.5z - 0.5z^2 \, . \tag{21}$$

Coefficient of low-pass filter can be obtained according to equation (26):

$$h_0 = 0.125, h_1 = 0.375, h_2 = 0.375, h_3 = 0.125 \, . \tag{22}$$

Coefficient of high-pass filter can be obtained according to equation (22):

$$g_1 = 0.5 \quad g_2 = -0.5 \, . \tag{23}$$

When spline function is selected as scaling function, algorithm of binary wavelet transformation for two-dimensional discrete shall be expressed in the following forms of discrete convolution:

$$a_{j+1}(n,m) = a_j * \overline{h_j}(n)\overline{h_j}(m) \, .$$

$$d^x_{j+1}(n,m) = a_j * [\overline{g_j}(n)\delta(m)] \, .$$

$$d^y_{j+1}(n,m) = a_j * [\delta(n)\overline{g_j}(m)] \, . \tag{24}$$

Above equation provides convenience for simulation of programming. a_{j+1} is result of a_j along horizontal and vertical low-pass filtering, while d^x_{j+1} and d^y_{j+1} separately are results of a_j along horizontal and vertical low-pass filtering. Therefore, the method of two-dimensional discrete convolution can be used to realize simulated program of algorithm proposed in the thesis.

3. Experimental analysis

3.1. Introduction to algorithm example

Structural model of certain steel arch bridge is shown in Fig. 1:

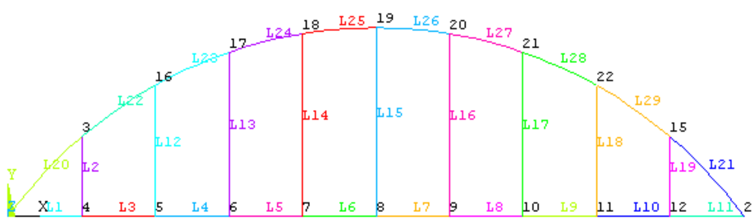


Fig. 1. Model and unit division diagram of steel arch bridge

Span of arch bridge is 100m; above-mentioned theory is applied to damage detection of bridge structure. Vibration response signal of structure can be obtained through applying shock load to node 7 in the model; on the basis, wavelet packet pulse response function and its wavelet packet energy spectrum is calculated. Two kinds of damages are defined in the structure: 1. 5% of damage for unit 3, 12, and

22 (which means that elasticity modulus of the three units reduces by 5%); II. 15% of damage for unit 3, 12, and 22 (which means that elasticity modulus of the three units reduces by 15%).

3.2. Result analysis

It is assumed that unit elasticity modulus reduces by 5% after structural damage. Damage 1 is that damage is in Area A (damage of unit 391, 392, 396, and 397). The method of wavelet transform is used to decompose dynamic response signal; obtained wavelet packet energy spectrum changes are shown in Fig. 2:

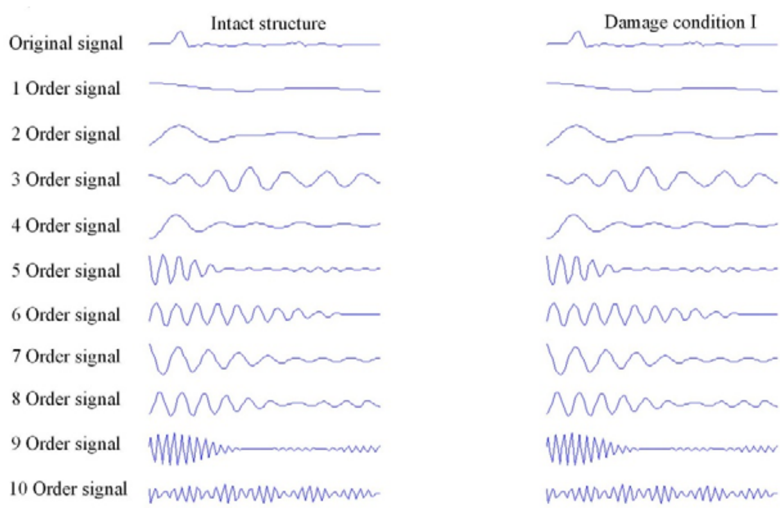
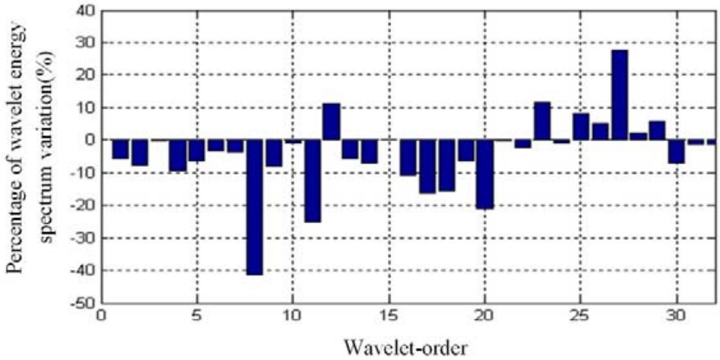


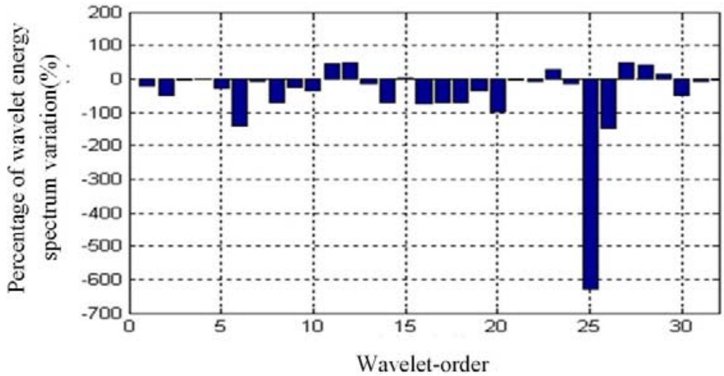
Fig. 2. Decomposition diagrams of wavelet transform signal for intact structure and damage condition I

Percentage (in Fig. 3) of energy spectrum variation for sub-signal before and after structural damage is solved. It is observed in Fig. 3 (a) that when unit damage is 5%, percentage of energy spectrum variation for sub-signal is fairly larger at 8 and 11 of wavelet-order. Meanwhile, it is observed in Fig. 3 (b) that when unit damage is 15%, percentage of energy spectrum variation for sub-signal is very obvious at 25 and 26 of wavelet-order. Obviously, the dimensionless percentage vector of the wavelet packet energy spectrum can be used to qualitatively and quantitatively describe structural damage conditions.

It can be observed from above results that no matter where the damage is, with increase of damage, differences of wavelet decomposition sub-signal between intact structure and damaged structure and variation of wavelet sub-signal energy spectrum are larger. It can be observed through comparing two damaged places that the closer the damaged area to with the two damaged places to the constraint, the larger the influence of damage on difference of wavelet decomposition sub-signal and variation of wavelet sub-signal energy spectrum.



(a) Damage condition I



(b) Damage condition II

Fig. 3. Percentage of wavelet energy spectrum variation under different conditions

4. Conclusion

In general, influences of damaged parts on inherent frequency of structure vary with the distance of it to the center. Damaged parts in the center have greater influences on inherent frequency of structure than that on the edge. However, these differences are very tiny. Response signal in structural transient analysis is subject to wavelet analysis in the thesis so as to determine wavelet sub-signal and wavelet packet energy spectrum between intact structure and damaged structure. Differences of structures after suffering tiny damage can be observed through variation of wavelet packet energy spectrum. When different parts of structure are identical, the farther the damaged place to the edge, the larger the damage and variation of wavelet packet energy spectrum are. Influence of damage on variation of constraint is obvious.

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